Mon. Not. R. astr. Soc. (1992) **254**, *Short Communication*, 15P–18P

Can weak localization of photons explain the opposition effect of Saturn's rings?

M. I. Mishchenko and J. M. Dlugach

The Main Astronomical Observatory of the Ukrainian Academy of Sciences, 252127 Kiev, Goloseevo, USSR

Accepted 1991 November 4. Received 1991 October 24

SUMMARY

Weak localization of photons in discrete disordered media (or the coherent backscattering mechanism) is shown to be a likely explanation of the opposition effect exhibited by Saturn's rings. Specifically, we assume that the particles of Saturn's rings are covered with small H_2O ice grains and compute theoretically the opposition effect produced by these grains via the coherent backscattering mechanism. Both the width and amplitude of the observed opposition effect at visible wavelengths are consistent with theoretical calculations for effective grain radii of about 0.1–1 μm. Such grains are known to be present in the outer B ring of Saturn and give rise to the so-called 'spokes'. Thus, we demonstrate that the opposition effect of Saturn's rings may be due to the surface properties of the individual ring particles rather than to interparticle shadowing, as is usually assumed.

1 INTRODUCTION

Photometric phase curves of many atmosphereless bodies in the Solar System exhibit a sharp peak when the phase angle approaches zero. Usually, the opposition brightening of the atmosphereless surfaces and Saturn's rings is attributed to the so-called shadowing mechanism (Irvine 1966; Morozhenko & Yanovitskij 1971; Lumme & Bowell 1981; Hapke 1986). Nevertheless, this shadowing mechanism, apparently, cannot appropriately reproduce the strong and narrow opposition spikes exhibited by icy satellites and the bright E-type asteroids 44 Nysa and 64 Angelina and seems not to be responsible for the opposition effect of Saturn's rings. The opposition spikes observed for these objects are very similar, which led Harris *et al.* (1989) to the conclusions that this remarkable opposition brightening is a normal property of moderate- to high-albedo atmosphereless surfaces, and that much of the opposition effect of the rings of Saturn may be due to the surface properties of the individual particles, rather than to interparticle shadowing, as has often been suggested. The latter conclusion was also drawn by Johnson *et al.* (1980), who observed unusual behaviour of the polarization phase curves of Saturn's rings near zero phase (see also Dollfus 1979).

The possible relevance of the so-called weak localization of photons in discrete disordered media (or coherent backscattering mechanism) to the opposition effect of atmosphereless surfaces was mentioned first by Kuga & Ishimaru (1984) and then by Shkuratov (1988), Hapke (1990) and Muinonen (1990). In this paper, we show that this weak localization of photons is a likely explanation of the opposition effect exhibited by the rings of Saturn. Specifically, we assume that the surfaces of the ring particles are covered with a layer of small H_2O ice grains and demonstrate that coherent backscattering of light from this surface layer can produce opposition spikes with a half-width of about $0\overset{\circ}{.}3$ and amplitudes of about 0.23 mag in the yellow and 0.28 mag in the blue (Franklin & Cook 1965).

2 CALCULATIONS

The theory of the coherent backscattering mechanism is basically well understood and is in excellent agreement with laboratory data (e.g., Nieto-Vesperinas & Dainty 1990; Sheng 1990). According to this theory, the multiply scattered radiation reflected by discrete disordered media is composed of two parts. The first part is the diffusely reflected background radiation, which comes from the sum of the so-called ladder terms of the Bethe–Salpeter equation. The second part is the coherent backscattering peak, which arises because a wave scattered through a certain multiple-scattering path can interfere with the wave scattered through the time-reversed path, the interference being constructive in the backscattering direction. This coherent part of the reflected radiation comes from the sum of the so-called cyclical terms of the Bethe–Salpeter equation (for terminology, see Tsang & Ishimuru 1985).

As was noted above, we assume that the surfaces of the particles of Saturn's rings are covered with a layer of small H_2O ice grains. Assuming this surface layer to be optically thick, we have for the half-width at half-maximum (HWHM) of the coherent backscattering peak (e.g., Stephen & Cwilich 1986; Wolf *et al.* 1988):

$$HWHM = \frac{\varepsilon\lambda}{2\pi\lambda_{tr}}, \tag{1}$$

where λ is the free-space wavelength, λ_{tr} is the transport mean free path and ε is a constant. Though the theoretically predicted inverse proportionality of $HWHM$ to λ_{tr} was confirmed in a lot of experiments, there is a scatter in the reported theoretical and experimental values of the factor ε. Nevertheless, all these values are close to a 'mean' value $\varepsilon = 0.5$, which, therefore, was used in all of our computations.

The theoretical calculation of the transport mean free path λ_{tr} for media composed of densely distributed grains of arbitrary size and shape is a very complicated problem which is still far from being solved. Therefore, in practical computations we had to use some simplifications. Specifically, we assumed that the grains are homogeneous, independently scattering spheres and used the following formula (Ishimaru 1978):

$$\lambda_{tr}^{-1} = nC_{ext}(1 - \omega\langle\cos\theta\rangle), \tag{2}$$

where n is the number of grains per unit volume, C_{ext} is the extinction cross-section, ω is the single-scattering albedo, and $\langle\cos\theta\rangle$ is the mean cosine of the scattering angle. To describe the distribution of the grains over radii, we used the standard gamma distribution (Hansen & Hovenier 1974):

$$n(r) = \text{constant} \times r^{(1-3v_{eff})/v_{eff}} \exp[-r/(r_{eff}v_{eff})], \tag{3}$$

where r_{eff} is the effective radius and v_{eff} is the effective variance. It is convenient to rewrite equation (2) as

$$\lambda_{tr}^{-1} = C_{ext}(1 - \omega\langle\cos\theta\rangle)\frac{3F}{4\pi r_{eff}^3}, \tag{4}$$

where F is the filling factor (i.e., the fraction of a volume occupied by the particles). From equations (1) and (4) it is obvious that $HWHM \to 0$ with $r_{eff} \to \infty$; also, for non-absorbing particles $HWHM \to 0$ with $r_{eff} \to 0$, while for absorbing particles $HWHM \to c$ with $r_{eff} \to 0$, where c is a constant which depends only on refractive index (*cf.* van de Hulst 1957; Bohren & Huffman 1983). Results of the corresponding Mie calculations are shown in Fig. 1 where $HWHM$ is plotted versus a dimensionless parameter $y = r_{eff}/\lambda$ for $F = 0.1$ and $v_{eff} = 0.05$. The real part of the refractive index is 1.31 (Warren 1984). Note that $HWHM$ is not significantly influenced by absorption which is modelled in Fig. 1 by using a small non-zero value of the imaginary part of the refractive index. Some absorption is necessary to match the estimated spherical albedos of the ring particles (e.g., Esposito *et al.* 1984).

It should be noted that, in principle, equation (4) is valid only in the limit $F \to 0$ (i.e., for independently scattering grains) and must be handled with care for large filling factors. Nevertheless, as is shown by the calculations of Wolf *et al.* (1988), this equation gives rather accurate results provided that the scattering grains are not too small as compared with the wavelength.

Thus, Fig. 1 evidently demonstrates that H_2O ice grains with effective radii greater than roughly 0.1 μm and smaller

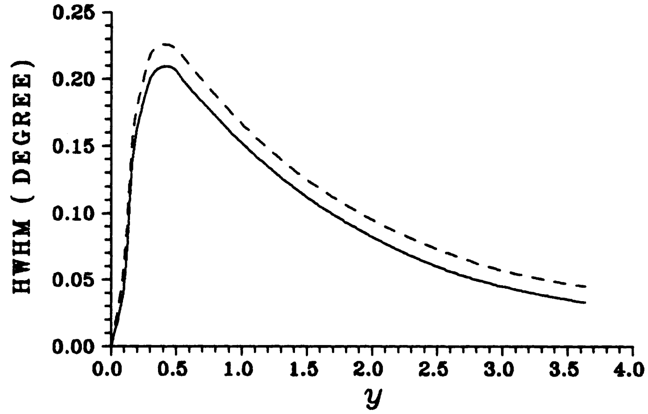

Figure 1. $HWHM$ versus a dimensionless parameter $y = r_{eff}/\lambda$ for the gamma distribution of spherical grains with $v_{eff} = 0.05$ and $F = 0.1$. The real part of the refractive index is 1.31. The imaginary part of the refractive index is 0 (solid line) and 0.003 (dashed line).

than roughly 1 μm, and filling factors from a reasonable range 0.1–0.3, (e.g., Hobs 1974) can reproduce the width (about 0°3) of the opposition spikes of Saturn's rings observed in the blue and yellow (Franklin & Cook 1965).

The second quantity that should be calculated theoretically and compared with the observational data, is the amplitude of the opposition spike ζ. This quantity is defined as the ratio of the total backscattered intensity at zero phase angle to the incoherent (diffuse) background intensity:

$$\zeta = \frac{I_{total}}{I_{background}} = \frac{I_{coherent} + I_{background}}{I_{background}}. \tag{5}$$

All the backscattered light can be divided into two parts. The first part is the light that is *singly reflected by unshadowed surface elements.* This backscattered light results from the multiple-scattering processes that occur inside individual unshadowed surface elements with sizes of the order of λ_{tr}. They both contribute to $I_{background}$ and give rise to the coherent backscattering peak with $HWHM$ given by equation (1). The second part is the light that is *multiply reflected by different surface elements.* These multiple reflections arise due to macroscopic surface roughness with scale much greater than λ_{tr} and due to successive reflections by different ring particles. This second part of the backscattered light contributes to $I_{background}$, but does not contribute to the coherent backscattering peak.

Precise calculations of both the singly and multiply reflected components of the backscattered intensity are difficult because they require detailed information about the large-scale surface structure and spatial distribution of the ring particles. The only thing that can be said now is that the observed amplitude of the opposition effect should be substantially smaller than the amplitude that is calculated for the light singly reflected by a macroscopically flat surface. To compute the latter amplitude, we used the rigorous vector theory which has been developed by Mishchenko (1991) and is summarized in the Appendix.* Note that, unlike $HWHM$,

*As was shown by Mishchenko & Dlugach (1991) for the case of unpolarized incident light, the scalar theory of weak localization can substantially overestimate the amplitude of the opposition effect and, therefore, should not be used in model computations.

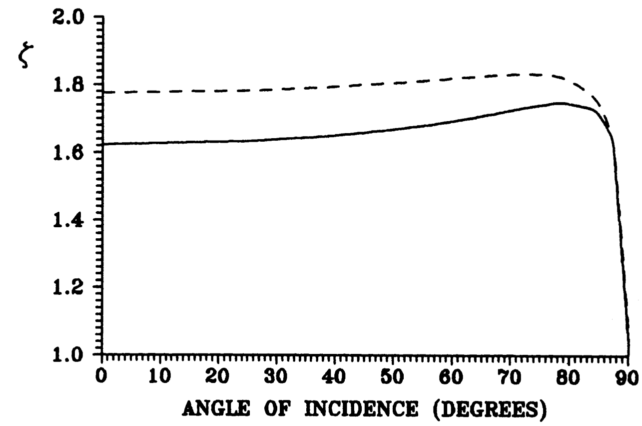

Figure 2. The amplitude of the opposition effect ζ versus the angle of incidence ϑ. The optically semi-infinite medium with macroscopically flat surface is composed of polydisperse spherical particles with $r_{\rm eff}/\lambda=1$ and $v_{\rm eff}=0.05$. The real part of the refractive index is 1.31. The imaginary part of the refractive index is 0 (solid line) and 0.003 (dashed line).

this amplitude does not depend on filling factor and depends only on size parameter and refractive index of scattering grains. Our calculations in the range $0.2 \leqslant y \leqslant 2$ have shown that the amplitude is practically independent of y. The typical dependence of the theoretically calculated amplitudes on the angle of incidence ϑ is shown in Fig. 2. Indeed, we see that these amplitudes of about 1.7 are substantially larger than the observed amplitudes of about 0.23–0.28 mag (Franklin & Cook 1965).

Finally we note that, as is shown by Fig. 2, the single-reflection amplitude tends to increase with increasing absorption. Also, due to multiple reflections, $I_{\rm background}$ must substantially decrease with increasing absorption (e.g., Lumme, Peltoniemi & Irvine 1990), while $I_{\rm coherent}$ is due to single reflections and is much less affected by absorption. Therefore, the observed amplitude of the opposition effect should increase with increasing absorption. Indeed, in accord with this conclusion, the yellow amplitude of the opposition effect of about 0.23 mag is smaller than the blue one of about 0.28 mag (Franklin & Cook 1965), the rings being darker in the blue than in the yellow (e.g., Esposito *et al.* 1984).

3 DISCUSSION

In this paper, we have shown that an upper surface layer which, apparently, covers particles of Saturn's rings and is composed of submicron-sized H_2O ice grains, can be responsible for the narrow and strong opposition effect exhibited by the rings of Saturn. This result is in agreement with recent investigations of Saturn's rings. The presence of submicron-sized H_2O ice grains in the outer B ring of Saturn has been demonstrated by the discovery of the so-called 'spokes' (e.g., Esposito *et al.* 1984). The fact that the spokes are dark at small phase angles and bright at large phase angles suggests that they are due to grains orders of magnitude finer than the ring particles. It is usually assumed that, as a result of various processes, dust grains covering the large ring particles become charged, are electrostatically levitated, and alter locally the scattering properties of the rings. As was found by Doyle, Dones & Cuzzi (1989), in spoke regions the phase function of the large particles is somewhat less for backscattering and the albedo is slightly smaller, consistent with a release of small dust grains from the surface of the large particles as a mechanism for spoke production. By studying colour *Voyager* imaging data of the outer B ring, Doyle & Grün (1990) concluded that effective grain radii are about 0.4–0.5 μm or greater and are likely to be very narrowly distributed at around 0.6 ± 0.2 μm. This result is in full agreement with our estimate. The possible origin of such grains is discussed, e.g., by Smoluchowski (1983).

ACKNOWLEDGMENTS

We are grateful to L. O. Kolokolova and L. R. Lisina for useful discussions.

REFERENCES

Bohren, C. F. & Huffman, D. R., 1983. *Absorption and Scattering of Light by Small Particles,* John Wiley, New York.

Chandrasekhar, S., 1950. *Radiative Transfer,* Clarendon Press, London.

de Rooij, W. A., 1985. *PhD thesis,* Free University, Amsterdam.

Dollfus, A., 1979. *Icarus,* **37,** 404.

Doyle, L. R. & Grün, E., 1990. *Icarus,* **85,** 168.

Doyle, L. R., Dones, L. & Cuzzi, J. N., 1989. *Icarus,* **80,** 104.

Esposito, L. W., Cuzzi, J. N., Holberg, J. B., Marouf, E. A., Tyler, G. L. & Porco, C. C., 1984. In: *Saturn,* p. 463, eds Gehrels, T. & Matthews, M. S., University of Arizona Press, Tucson.

Franklin, F. A. & Cook, A. F., 1965. *Astr. J.,* **70,** 704.

Hansen, J. E. & Hovenier, J. W., 1974. *J. atmos. Sci.,* **31,** 1137.

Hapke, B., 1986. *Icarus,* **67,** 264.

Hapke, B., 1990. *Icarus,* **88,** 407.

Harris, A. W. *et al.,* 1989. *Icarus,* **81,** 365.

Hobs, P., 1974. *Ice Physics,* Clarendon Press, London.

Hovenier, J. W. & van der Mee, C. V. M., 1983. *Astr. Astrophys.,* **128,** 1.

Irvine, W. M., 1966. *J. geophys. Res.,* **71,** 2931.

Ishimaru, A., 1978. *Wave Propagation and Scattering in Random Media,* Academic Press, New York.

Johnson, P. E., Kemp, J. C., King, R. & Barbour, M. S., 1980. *Nature,* **283,** 146.

Kuga, Y. & Ishimaru, A., 1984. *J. opt. Soc. Am. A,* **1,** 831.

Lumme, K. & Bowell, E., 1981. *Astr. J.,* **86,** 1694.

Lumme, K. Peltoniemi, J. I. & Irvine, W. M., 1990. *Transport Theory & Statist. Phys.,* **19,** 317.

Mishchenko, M. I., 1991. *Phys. Rev. B,* in press.

Mishchenko, M. I. & Dlugach, J. M., 1991. *Astrophys. Space Sci.,* in press.

Morozhenko, A. V. & Yanovitskij, E. G., 1971. *Astr. Zh.,* **48,** 172.

Muinonen, K., 1990. *PhD thesis,* University of Helsinki.

Nieto-Vesperinas, M. & Dainty, J. C., eds, 1990. *Scattering in Volumes and Surfaces,* North-Holland, Amsterdam.

Prishivalko, A. P., Babenko, V. A. & Kuz'min, V. N., 1984. *Scattering and Absorption of Light by Inhomogeneous and Anisotropic Spherical Particles,* Nauka i Tekhnika, Minsk. In Russian.

Sheng, P., ed., 1990. *Scattering and Localization of Classical Waves in Random Media,* World Scientific, Singapore.

Shkuratov, Yu. G., 1988. *Kinem. Fiz. Nebes. Tel,* **4,** No. 4, 33.

Smoluchowski, R., 1983. *Icarus,* **54,** 263.

Stephen, M. J. & Cwilich, G., 1986. *Phys. Rev. B,* **34,** 7564.

Tsang, L. & Ishimaru, A., 1985. *J. opt. Soc. Am. A,* **2,** 2187.

van de Hulst, H. C., 1957. *Light Scattering by Small Particles,* John Wiley, New York.

Warren, S., 1984. *Appl. Opt.,* **23,** 1206.

Wolf, P. E., Maret, G., Akkermans, E. & Maynard, R., 1988. *J. Phys. France,* **49,** 63.

APPENDIX

In this Appendix, we briefly summarize the method that was used to compute the amplitude of the opposition effect due to single reflection of light from discrete random media with macroscopically flat surfaces. Let **S** be the total Stokes backscattering matrix in the common (I, Q, U, V)-representation of polarized light (van de Hulst 1957). We write

$$\mathbf{S}=\mathbf{S}^{1}+\mathbf{S}^{\mathrm{L}}+\mathbf{S}^{\mathrm{C}}, \tag{A1}$$

where 1 denotes the contribution of the first-order scattering, L denotes the contribution of all the other ladder terms, and C denotes the contribution of all the cyclical terms. Assuming that the scattering medium is composed of randomly positioned, independent, discrete scatterers, one can express the elements of the matrix $\mathbf{S}^{\mathrm{C}}$ in the elements of the matrix $\mathbf{S}^{\mathrm{L}}$ as (Mishchenko 1991)

$$\mathbf{S}^{\mathrm{C}}=\begin{bmatrix} S^{\mathrm{C}}_{11} & S^{\mathrm{L}}_{12} & 0 & 0 \\ S^{\mathrm{L}}_{12} & S^{\mathrm{C}}_{22} & 0 & 0 \\ 0 & 0 & S^{\mathrm{C}}_{33} & S^{\mathrm{L}}_{34} \\ 0 & 0 & -S^{\mathrm{L}}_{34} & S^{\mathrm{C}}_{44} \end{bmatrix}, \tag{A2}$$

where

$$\begin{aligned} S^{\mathrm{C}}_{11}&=\tfrac{1}{2}[S^{\mathrm{L}}_{11}+S^{\mathrm{L}}_{22}-S^{\mathrm{L}}_{33}+S^{\mathrm{L}}_{44}],\\ S^{\mathrm{C}}_{22}&=\tfrac{1}{2}[S^{\mathrm{L}}_{11}+S^{\mathrm{L}}_{22}+S^{\mathrm{L}}_{33}-S^{\mathrm{L}}_{44}],\\ S^{\mathrm{C}}_{33}&=\tfrac{1}{2}[-S^{\mathrm{L}}_{11}+S^{\mathrm{L}}_{22}+S^{\mathrm{L}}_{33}+S^{\mathrm{L}}_{44}],\\ S^{\mathrm{C}}_{44}&=\tfrac{1}{2}[S^{\mathrm{L}}_{11}-S^{\mathrm{L}}_{22}+S^{\mathrm{L}}_{33}+S^{\mathrm{L}}_{44}]. \end{aligned} \tag{A3}$$

Equations (A2) and (A3) hold for randomly oriented particles of any size, shape, and refractive index. From these equations, we have for the amplitude of the opposition effect:

$$\zeta=\frac{[S^{1}_{11}+S^{\mathrm{L}}_{11}+S^{\mathrm{C}}_{11}]}{[S^{1}_{11}+S^{\mathrm{L}}_{11}]}=\frac{[S^{1}_{11}+S^{\mathrm{L}}_{11}+\frac{1}{2}(S^{\mathrm{L}}_{11}+S^{\mathrm{L}}_{22}-S^{\mathrm{L}}_{33}+S^{\mathrm{L}}_{44})]}{[S^{1}_{11}+S^{\mathrm{L}}_{11}]}. \tag{A4}$$

It is well known (e.g., Prishivalko, Babenko & Kuz'min 1984 and references therein) that the Bethe–Salpeter equation under the ladder approximation of independent discrete scatterers results in the common vector radiative transfer equation (Chandrasekhar 1950; Hovenier & van der Mee 1983). Therefore, by solving this radiative transfer equation numerically, we can calculate the contributions $\mathbf{S}^{1}$ and $\mathbf{S}^{\mathrm{L}}$ and then determine the amplitude of the opposition effect ζ from equation (A4) for particular scattering models. In the computations reported, we used the computational procedures that have been proposed and extensively described by de Rooij (1985).